Sydney Technical High School

Student Name ____________________________________

Teacher's Name: ____________________________________

Extension 1 Mathematics

TRIAL HSC

August 2022

General Instructions

- Reading time – 10 minutes
- Working time – 120 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt questions 11-14
- Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

---

1. The number of different arrangements of the letters of the word NORTHERN which begin and end with the letter N is:

   A) $\frac{8!}{2!}$

   B) $\frac{6!}{2!}$

   C) $\frac{8!}{2!2!}$

   D) $\frac{6!}{2!2!}$

2. Given $f(x) = \sqrt{x} - 1$, what are the domain and range of $f^{-1}(x)$ respectively?

   A) $x \geq 0, y \geq -1$

   B) $x \geq -1, y \geq -1$

   C) $x \geq -1, y \geq 0$

   D) $x \geq 1, y \geq 0$

3. Find $\lim_{x \to 0} \frac{sin\frac{x}{3}}{5x}$

A) $\frac{1}{15}$

B) $\frac{3}{5}$

C) $\frac{5}{3}$

D) 15

4. Which of the following is the primitive of $\frac{1}{\sqrt{4-9x^2}}$?

A) $\frac{1}{2}sin^{-1}(3x) + c$

B) $\frac{1}{3}sin^{-1}(3x) + c$

C) $\frac{1}{3}sin^{-1}\left(\frac{2x}{3}\right) + c$

D) $\frac{1}{3}sin^{-1}\left(\frac{3x}{2}\right) + c$

5. The polynomial $P(x) = 8x^3 + ax^2 - 4x + 1$ has a factor of $2x + 1$.

What is the value of a?

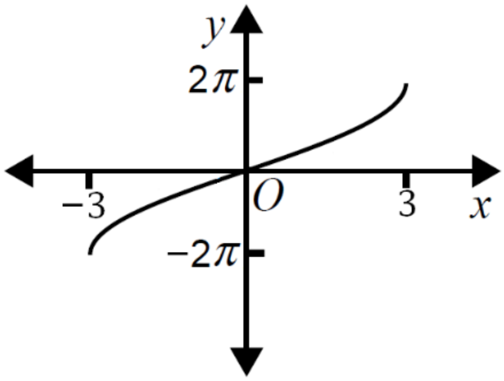
A) -8

B) 0

C) 3

D) 8

6. The diagram below shows the graph of a function



A possible equation for the function is:

A) $y = \frac{1}{4} sin^{-1}(3x)$

B) $y = \frac{1}{4} sin^{-1}\left(\frac{x}{3}\right)$

C) $y = 4sin^{-1}\left(\frac{x}{3}\right)$

D) $y = 4sin^{-1}(3x)$

7. Given the points $A(1, 3)$, $B(4, 5)$ and $C(2, r)$, it is known that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$.

What is the value of r?

A) -8

B) -3

C) 3

D) 8

8. A Bernoulli variable, X, has a value of p such that $E(X) = 5Var(X)$.

Given that $p \neq 0$, what is the value of p?

A) $\frac{1}{2}$

B) $\frac{4}{5}$

C) $\frac{1}{5}$

D) $\frac{3}{5}$

9. Suppose that $f(x)$ is a continuous function and that $\int_1^5 f(x)dx = -6$ and $\int_2^5 3f(x)dx = 6$.

What is the value of $\int_1^2 f(x)dx$?

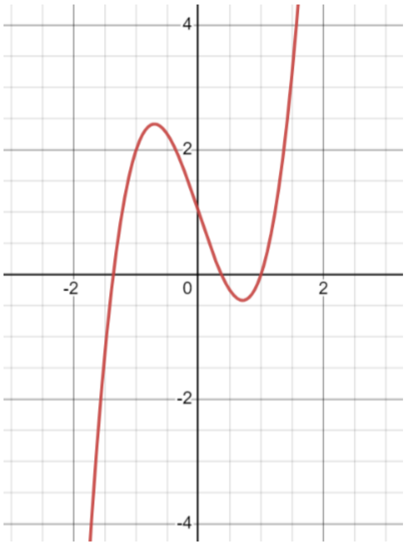
A) -8

B) -12

C) 8

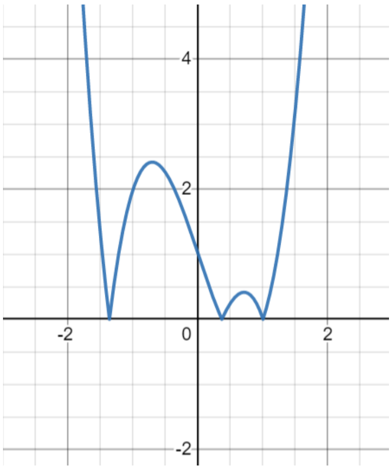
D) 12

10. The graph of the function $f(x)$ is drawn below

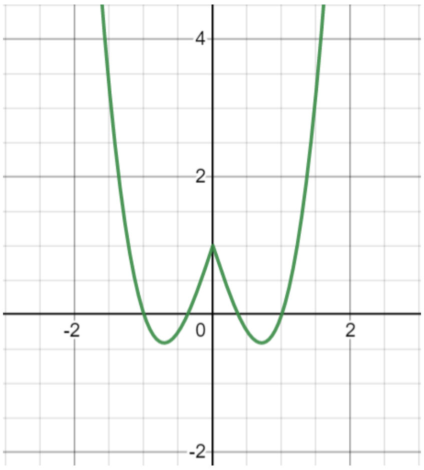


Which of the following best represents the graph of $y = |f(|x|)|$?

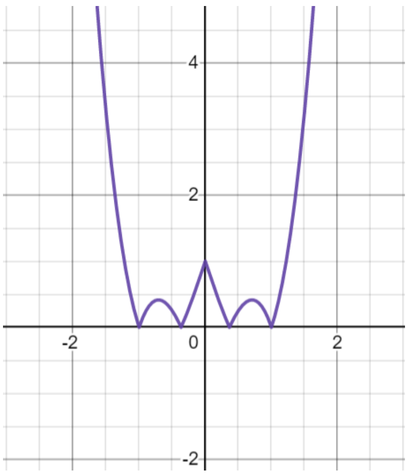
A)



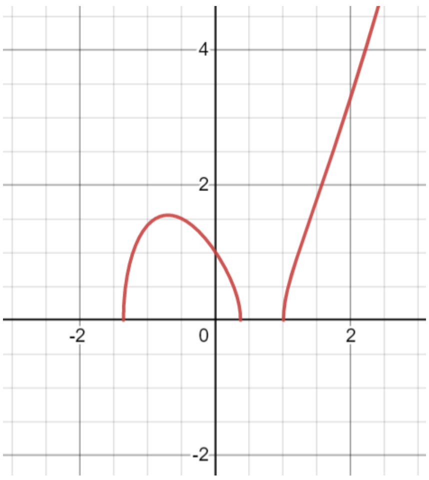
B)



C)



D)



End of Section I

SECTION II

60 marks

Attempt Questions 11-14.

Allow about 1 hour and 45 minutes for this section.

Answer each question on a new page in the answer booklet.

In questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a NEW page.**

a) Solve for x: $\frac{3}{x-1} \geq 2$ 3

b) Find the value of $sin 15°$ in simplest exact form 2

c) Find the Cartesian equation for the function with these parametric equations: 1

$$x = 2t + 1$$

$$y = t - 2$$

d) A committee of five is to be chosen from six men and seven women.

(i) How many committees are possible if there are no restrictions? 1

(ii) How many committees are possible if there are more women than men? 2

Question 11 continues on page 9

e) A rock drops into a lake, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 6 cm/s. 2

At what rate is the area enclosed within the ripple increasing when the radius is 12 cm?

f) (i) Write $\sqrt{3}cos\theta - sin\theta$ in the form $Rcos(\theta + \alpha)$ 2

(ii) Hence, or otherwise, solve $\sqrt{3}cos\theta - sin\theta = 1$ for $0 \leq \theta \leq 2\pi$ 2

Question 12 (15 marks) **Start a NEW page.**

a) Find the exact value of $sin\left(2cos^{-1}\frac{2}{3}\right)$ 2

b) The polynomial $P(x) = ax^3 + bx^2 + c$ has a double root at $x = 3$ and has remainder -36 when divided by $x + 3$. 3

Find the values of a, b and c.

c) Use the substitution $u = x - 3$ to evaluate 3

$\int_3^4 x\sqrt{x-3}\, dx$

d) Find the term independent of x in the expansion of $\left(3x^4 - \frac{1}{x^2}\right)^9$ 3

e) Prove by mathematical induction that $7^n - 3^n$ is divisible by 4 for $n \geq 1$ 3

f) State the range of $y = cos^{-1}(cosx)$ 1

Question 13 (15 marks) **Start a NEW page.**

a) Consider the points $A(2,-2)$ and $B(2,6)$. Using vector methods or otherwise, show that $\angle AOB = 117°$ to the nearest degree, where O is the origin. 2

b) A container of water, heated to $100°C$, is placed in a cool room where the temperature is maintained at a constant $-5°C$.

After t minutes, the rate of change of the temperature, $T°C$ of the water is given by $\frac{dT}{dt} = -k(T+5)$, where k is a constant.

(i) Assuming the function $T = Ae^{-kt} - 5$, where A is a constant, is a solution to the above differential equation, find the value of A. 1

(ii) After 30 minutes, the water temperature falls to $20°C$. 2

Find, to the nearest degree, the water temperature after a further 10 minutes.

c) Jürgen Klopp enters a football tipping competition. The probability that he chooses the winner of any one game is 0.7. In a competition where there are 9 games in a round:

(i) What is the probability that he will choose exactly seven winners? 1

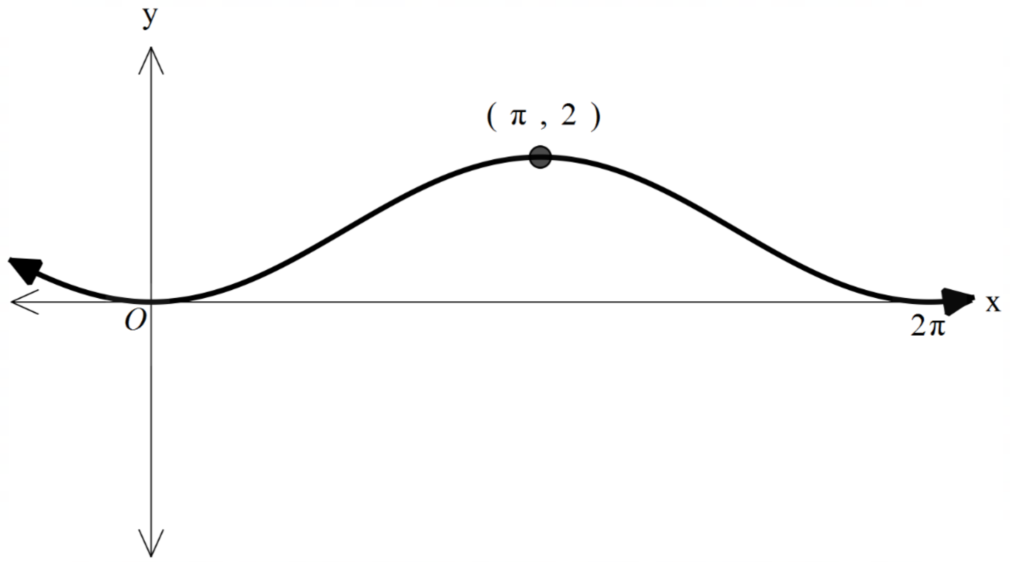
(ii) What is the probability that he will choose less than seven winners? 2

Question 13 continues on page 12

d) (i) Find $\frac{d}{dx}(x\tan^{-1}x)$ 1

(ii) Hence, find $\int_0^1 \tan^{-1}x\, dx$, leaving your answer in exact form 3

e) The diagram below shows part of the graph $y = 1 - \cos x$.



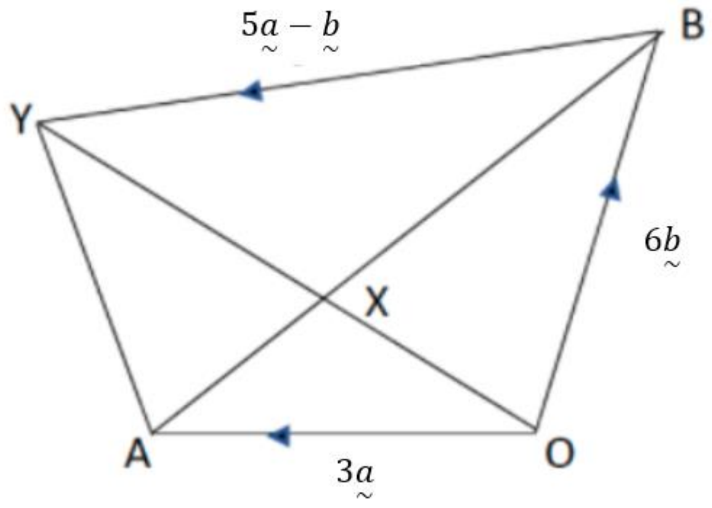
Find the volume generated when the area bounded by $y = 1 - \cos x$, $x = \frac{\pi}{2}$, 3

$x = \frac{3\pi}{2}$ and the x-axis is rotated about the x-axis.

Leave your answer in exact form.

Question 14 (15 marks) **Start a NEW page.**

a)



X is the point on AB such that $AX{:}XB = 1{:}2$ and $\overrightarrow{BY} = 5\underset{\sim}{a} - \underset{\sim}{b}$. $\overrightarrow{OA} = 3\underset{\sim}{a}$ and $\overrightarrow{OB} = 6\underset{\sim}{b}$.

(i) Express $\overrightarrow{AB}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ 1

(ii) Hence or otherwise, prove $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$ 2

Question 14 continues on page 14

b) Samsung does a quality check of their latest television model. In a sample of 160 televisions, 8 were found to be defective.

(i) It is known that the sample proportion is approximately normally distributed. Show that the sample mean is 0.05 and the sample standard deviation is 0.01723. 2

(ii) The Hilton group needs to purchase 160 televisions for a new hotel. By referring to the z-score table provided, estimate the probability that the number of defective televisions purchased is at least 4 but no more than 6. 3

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189

Question 14 continues on page 15

c) (i) Show that $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ has no stationary points. 2

(ii) Given that $y = \pm 1$ are horizontal asymptotes, sketch the curve. 1

(iii) For $k > 0$, consider the area enclosed by the curve, the lines $y = 1$, $x = 0$ and $x = k$. 2

Show that this area can be expressed in the form $ln\left(\frac{2e^k}{e^k + e^{-k}}\right)$

(iv) Hence, deduce that for all values of k, the area found in part (iii) is always less than $ln2$. 2

End of paper

Student Name: ______________________ Teacher Name: ______________________

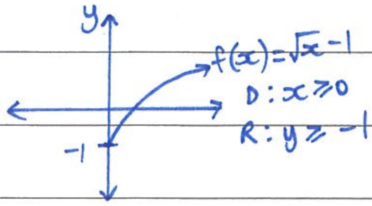
Extension 1 Trial HSC 2022 Solutions

Multiple choice

1. B
2. C
3. A
4. D
5. A
6. C
7. D
8. B
9. A
10. C

1. $\frac{6!}{2!}$ (B)

2. $f(x)=\sqrt{x}-1$



$\therefore$ For $y=f^{-1}(x)$,

$D: x \geq -1$

$R: y \geq 0$

(C)

3. $\lim_{x\to 0} \frac{\sin\frac{x}{3}}{5x} = \lim_{x \to 0} \frac{\sin\frac{x}{3}}{\frac{x}{3}} \times \frac{1}{3} \times \frac{1}{5}$

$= \frac{1}{15}$

(A)

4. $\int \frac{1}{\sqrt{4-9x^2}}\,dx = \int \frac{1}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{\frac{4}{9}-x^2}}\,dx$

$= \frac{1}{3}\sin^{-1}\left(\frac{x}{\left(\frac{2}{3}\right)}\right) + C$

$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$

(D)

Student Name: ______________________ Teacher Name: ______________________

5. $P(-\frac{1}{2})=0$

$8(-\frac{1}{2})^3+(-\frac{1}{2})^2a-4(-\frac{1}{2})+1=0$

$2+\frac{1}{4}a=0$

$\frac{1}{4}a=-2$

$a=-8$

(A)

6. For $y=\sin^{-1}x$, $D: -1\leq x\leq 1$, $R: -\frac{\pi}{2}\leq y\leq\frac{\pi}{2}$

$\frac{1}{3}\times$ horizontal dilation $4\times$ vertical dilation

$y=4\sin^{-1}\left(\frac{x}{3}\right)$, $D: -3\leq x\leq 3$, $R: -2\pi\leq y\leq 2\pi$

(C)

7. $\overrightarrow{AB}=\begin{pmatrix}3\\2\end{pmatrix}$ $\overrightarrow{BC}=\begin{pmatrix}-2\\r-5\end{pmatrix}$ Perpendicular so $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$

$\overrightarrow{AB}\cdot\overrightarrow{BC}=\begin{pmatrix}3\\2\end{pmatrix}\cdot\begin{pmatrix}-2\\r-5\end{pmatrix}$

$=-6+2r-10$

$-6+2r-10=0$

$2r=16$

$r=8$

(D)

8. $E(X)=5\,Var(X)$ $p\neq 0$

$p=5p(1-p)$

$p=5p-5p^2$

$5p^2-4p=0$

$p(5p-4)=0$

$p=\frac{4}{5}$ as $p\neq 0$

(B)

9. $\int_1^5 f(x)\,dx=\int_1^2 f(x)\,dx+3\int_2^5 f(x)\,dx$

$-6=\int_1^2 f(x)\,dx+2$

$\therefore \int_1^2 f(x)\,dx=-8$

(A)

10. (C)

Student Name: ______________________ Teacher Name: ______________________

Question 11

a) $\dfrac{3(x-1)^2}{x-1} \geq 2(x-1)^2 \quad x \neq 1$

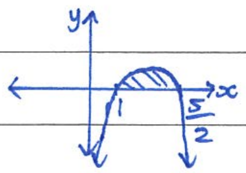
$3(x-1) \geq 2(x-1)^2$

$3(x-1) - 2(x-1)^2 \geq 0$

$(x-1)[3-2(x-1)] \geq 0$

$(x-1)(3-2x+2) \geq 0$

$(x-1)(-2x+5) \geq 0$



$\therefore 1 < x \leq \frac{5}{2}$

b) $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \begin{matrix} \times\sqrt{2} \\ \times\sqrt{2} \end{matrix}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

c) $x = 2t+1$ —① $\quad y = t-2$

$t = y+2$

sub into ①

$x = 2(y+2)+1$

$x = 2y+4+1$

$x - 2y - 5 = 0$

Student Name: ______________________ Teacher Name: ______________________

11. d) (i) $^{13}C_5 = 1287$

(ii) 3, 4 or 5 women

$^7C_3 \times {^6C_2} + {^7C_4} \times {^6C_1} + {^7C_5} \times {^6C_0}$

$= 756$

e) $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$ $\quad$ $\frac{dr}{dt} = 6 \text{ cm/s}$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 2\pi r \times 6$

$= 12\pi r$

when $r = 12$, $\frac{dA}{dt} = 12\pi \times 12$

$= 144\pi \text{ cm}^2/\text{s}$

f) (i) $\sqrt{3}\cos\theta - \sin\theta = R\cos(\theta + \alpha)$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$R\cos\alpha = \sqrt{3}$ — ① $\quad$ $R\sin\alpha = 1$ — ②

② ÷ ①

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$R^2 = (\sqrt{3})^2 + 1^2$

$R^2 = 4$

$R = 2$, $R > 0$

$\therefore \sqrt{3}\cos\theta - \sin\theta = 2\cos\left(\theta + \frac{\pi}{6}\right)$

(ii) $2\cos\left(\theta + \frac{\pi}{6}\right) = 1$

$\cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}$ $\quad$ $\frac{\pi}{6} \leq \theta + \frac{\pi}{6} \leq \frac{13\pi}{6}$

related angle $= \frac{\pi}{3}$

$\theta + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore \theta = \frac{\pi}{6}, \frac{3\pi}{2}$

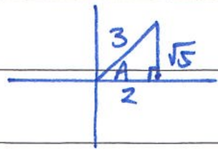
Student Name: ______________________ Teacher Name: ______________________

Question 12

a) $\sin\left(2\cos^{-1}\frac{2}{3}\right)$ Let $\cos^{-1}\frac{2}{3} = A$

$\cos A = \frac{2}{3}$

$\sin A = \frac{\sqrt{5}}{3}$



$\sin\left(2\cos^{-1}\frac{2}{3}\right) = \sin 2A$

$= 2\sin A\cos A$

$= 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$

$= \frac{4\sqrt{5}}{9}$

b) $P(x) = ax^3 + bx^2 + c$

$P'(x) = 3ax^2 + 2bx$

$P(3) = P'(3) = 0$ $P(-3) = -36$

$P(3) = 27a + 9b + c$

$27a + 9b + c = 0$ — ①

$P'(3) = 27a + 6b$

$27a + 6b = 0$ — ②

$P(-3) = -27a + 9b + c$

$-27a + 9b + c = -36$ — ③

① − ③

$27a + 9b + c = 0$

$-$ $-27a + 9b + c = -36$

$54a = 36$

$a = \frac{2}{3}$

sub into ②

$27\left(\frac{2}{3}\right) + 6b = 0$

$18 + 6b = 0$

$b = -3$

sub into ①

$27\left(\frac{2}{3}\right) + 9(-3) + c = 0$

$c = 9$

$\therefore a = \frac{2}{3}, b = -3, c = 9$

Student Name: ______________________________ Teacher Name: ______________________________

12.c) $\int_3^4 x\sqrt{x-3}\,dx$

$u = x-3$

$\frac{du}{dx} = 1$

$du = dx$

when $x=4, u=1$

when $x=3, u=0$

$= \int_0^1 (u+3)\sqrt{u}\,du$

$= \int_0^1 (u+3)u^{\frac{1}{2}}\,du$

$= \int_0^1 u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\,du$

$= \left[\frac{2u^{\frac{5}{2}}}{5} + \frac{3u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= \left[\frac{2u^{\frac{5}{2}}}{5} + 2u^{\frac{3}{2}}\right]_0^1$

$= \left[\frac{2(1)^{\frac{5}{2}}}{5} + 2(1)^{\frac{3}{2}}\right] - 0$

$= \frac{12}{5}$

d) $\left(3x^4 - \frac{1}{x^2}\right)^9$

General term $= {}^9C_r\,(3x^4)^{9-r}(-x^{-2})^r$

$= {}^9C_r\,(3)^{9-r}x^{36-4r}(-1)^r x^{-2r}$

$= {}^9C_r\,(3)^{9-r}(-1)^r x^{36-6r}$

Term independent of x: $36-6r=0$

$r=6$

$\therefore$ Term independent of x is ${}^9C_6(3)^{9-6}(-1)^6$

$= 2268$

Student Name: ______________________ Teacher Name: ______________________

12.e) Show true for $n=1$,

$7^1 - 3^1 = 4$

which is divisible by 4

$\therefore$ true for $n=1$

Assume true for $n=k$, $k \in \mathbb{Z}^+$

i.e. $\frac{7^k - 3^k}{4} = M$ where M is an integer.

$7^k - 3^k = 4M$

Prove true for $n=k+1$,

i.e. Prove $7^{k+1} - 3^{k+1}$ is divisible by 4.

$7^{k+1} - 3^{k+1} = 7^k . 7 - 3^{k+1}$

$= 7(4M + 3^k) - 3^k . 3$ using assumption

$= 28M + 7.3^k - 3.3^k$

$= 28M + 4.3^k$

$= 4(7M + 3^k)$ which is divisible by 4.

$\therefore$ If the statement is true for $n=k$, it is also true for $n=k+1$. As it is true for $n=1$, it will be true for $n=2, 3, 4$ and so on, i.e. it is true for all positive integers n.

f) $y = \cos^{-1}(\cos x)$

Range of $y = \cos^{-1} x$

$0 \le y \le \pi$

Student Name: ______________________ Teacher Name: ______________________

Question 13

a) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$|\overrightarrow{OA}| = \sqrt{2^2+(-2)^2}$ $|\overrightarrow{OB}| = \sqrt{2^2+6^2}$

$= \sqrt{8}$ $= \sqrt{40}$

$$\cos \angle AOB = \frac{\begin{pmatrix} 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix}}{\sqrt{8} \times \sqrt{40}}$$

$$= \frac{2 \times 2 + (-2) \times 6}{\sqrt{320}}$$

$$= \frac{-8}{\sqrt{320}}$$

$\therefore \angle AOB = 116° 34'$

$= 117°$ (nearest degree)

b)(i) $T = Ae^{-kt} - 5$ when $t=0, T=100$

$100 = Ae^0 - 5$

$A = 105$

(ii) $T = 105e^{-kt} - 5$

when $t=30, T=20$

$20 = 105e^{-30k} - 5$

$25 = 105e^{-30k}$

$\frac{5}{21} = e^{-30k}$

$\ln\left(\frac{5}{21}\right) = -30k$

$k = -\frac{1}{30}\ln\left(\frac{5}{21}\right)$

$\therefore T = 105e^{-t\left(-\frac{1}{30}\ln\left(\frac{5}{21}\right)\right)} - 5$

when $t=40$, $T = 105e^{-40\left(-\frac{1}{30}\ln\left(\frac{5}{21}\right)\right)} - 5$

$= 10.49\ldots$

$= 10°C$ (nearest degree)

Student Name: ______________________ Teacher Name: ______________________

13. c)(i) P(exactly 7 winners) $= {}^9C_7(0\cdot7)^7(0\cdot3)^2$

$= 0\cdot2668$ (4 d.p.)

(ii) P(less than 7 winners) = 1 − P(7 winners) − P(8 winners) − P(9 winners)

$= 1 - {}^9C_7(0\cdot7)^7(0\cdot3)^2 - {}^9C_8(0\cdot7)^8(0\cdot3)^1 - {}^9C_9(0\cdot7)^9(0\cdot3)^0$

$= 0\cdot5372$ (4 d.p.)

d)(i) $\frac{d}{dx}(x\tan^{-1}x) = \tan^{-1}x + \frac{x}{1+x^2}$

(ii) $\int \frac{d}{dx}(x\tan^{-1}x)\,dx = \int \tan^{-1}x\,dx + \int \frac{x}{1+x^2}\,dx$

$\therefore \int_0^1 \tan^{-1}x\,dx = [x\tan^{-1}x]_0^1 + \int_0^1 \frac{x}{1+x^2}\,dx$

$= [x\tan^{-1}x]_0^1 - \frac{1}{2}[\ln(1+x^2)]_0^1$

$= [1\tan^{-1}1 - 0\tan^{-1}0] - \frac{1}{2}[\ln(1+1) - \ln(1+0)]$

$= \frac{\pi}{4} - \frac{1}{2}[\ln 2 - 0]$

$= \frac{\pi}{4} - \frac{1}{2}\ln 2$

e) $V = \pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1-\cos x)^2\,dx$

$= \pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 - 2\cos x + \cos^2 x\,dx$

$= \pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 - 2\cos x + \frac{1}{2} + \frac{1}{2}\cos 2x\,dx$

$= \pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{3}{2} - 2\cos x + \frac{1}{2}\cos 2x\,dx$

$= \pi\left[\frac{3}{2}x - 2\sin x + \frac{1}{4}\sin 2x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= \pi\left[\left(\frac{3}{2}\times\frac{3\pi}{2} - 2\sin\frac{3\pi}{2} + \frac{1}{4}\sin 3\pi\right) - \left(\frac{3}{2}\times\frac{\pi}{2} - 2\sin\frac{\pi}{2} + \frac{1}{4}\sin\pi\right)\right]$

$= \pi\left[\left(\frac{9\pi}{4} - 2(-1) + 0\right) - \left(\frac{3\pi}{4} - 2 + 0\right)\right]$

$= \pi\left[\frac{3\pi}{2} + 4\right]$

$= \frac{3\pi^2}{2} + 4\pi$ $\quad\therefore$ Volume is $\frac{3\pi^2}{2} + 4\pi$ units^3

Student Name: ______________________ Teacher Name: ______________________

Question 14

a)(i) $\vec{OA} + \vec{AB} = \vec{OB}$

$3\underset{\sim}{a} + \vec{AB} = 6\underset{\sim}{b}$

$\vec{AB} = 6\underset{\sim}{b} - 3\underset{\sim}{a}$

(ii) $\vec{AX} = \frac{1}{3}\vec{AB}$ as $AX : XB = 1 : 2$

$= \frac{1}{3}(6\underset{\sim}{b} - 3\underset{\sim}{a})$

$= 2\underset{\sim}{b} - \underset{\sim}{a}$

$\vec{OX} = \vec{OA} + \vec{AX}$

$= 3\underset{\sim}{a} + 2\underset{\sim}{b} - \underset{\sim}{a}$

$= 2\underset{\sim}{a} + 2\underset{\sim}{b}$

$\vec{OY} = \vec{OA} + \vec{AY}$ and $\vec{AY} = \vec{AB} + \vec{BY}$

$= 3\underset{\sim}{a} + 5\underset{\sim}{b} + 2\underset{\sim}{a}$ $\quad$ $= 6\underset{\sim}{b} - 3\underset{\sim}{a} + 5\underset{\sim}{a} - \underset{\sim}{b}$

$= 5\underset{\sim}{a} + 5\underset{\sim}{b}$ $\quad$ $= 5\underset{\sim}{b} + 2\underset{\sim}{a}$

Now $\frac{2}{5}\vec{OY} = \frac{2}{5}(5\underset{\sim}{a} + 5\underset{\sim}{b})$

$= 2\underset{\sim}{a} + 2\underset{\sim}{b}$

$= \vec{OX}$

$\therefore \vec{OX} = \frac{2}{5}\vec{OY}$ as required.

b)(i) $\hat{p} = \frac{8}{160}$

$= 0.05$

$\mu_{\hat{p}} = \frac{n \times p}{n}$ $\quad$ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$

$= \frac{160 \times 0.05}{160}$ $\quad$ $= \sqrt{\frac{0.05 \times 0.95}{160}}$

$= 0.05$ $\quad$ $= 0.01723$ (5 d.p.)

Student Name: ______________________ Teacher Name: ______________________

14.b)(ii) If 4 TV's defective: $\hat{p} = \frac{4}{160}$

$= 0.025$

$z = \frac{0.025 - 0.05}{0.01723}$

$= -1.45$ (2d.p.)

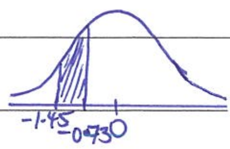
If 6 TV's defective: $\hat{p} = \frac{6}{160}$

$= 0.0375$

$z = \frac{0.0375 - 0.05}{0.01723}$

$= -0.73$ (2d.p.)

$\therefore P(4 < X < 6) = P(-1.45 < z < -0.73)$



$= P(z < -0.73) - P(z < -1.45)$

$= [1 - P(z < 0.73)] - [1 - P(z < 1.45)]$

$= [1 - 0.76730] - [1 - 0.92647]$

$= 0.15917$

c)(i) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\frac{dy}{dx} = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$

$= \frac{e^{2x} + e^x . e^{-x} + e^{-x} . e^x + e^{-2x} - [e^{2x} - e^x . e^{-x} - e^{-x} . e^x + e^{-2x}]}{(e^x + e^{-x})^2}$

$= \frac{e^{2x} + 1 + 1 + e^{-2x} - [e^{2x} - 1 - 1 + e^{-2x}]}{(e^x + e^{-x})^2}$

$= \frac{4}{(e^x + e^{-x})^2}$

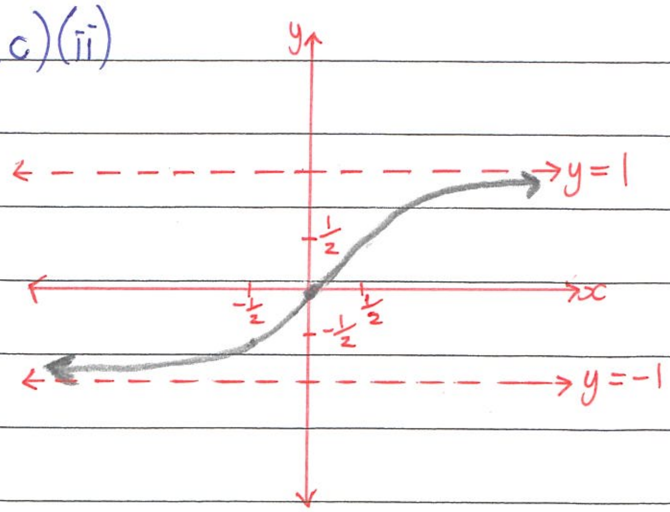
Stationary points occur when $\frac{dy}{dx} = 0$

i.e. $\frac{4}{(e^x + e^{-x})^2} = 0$

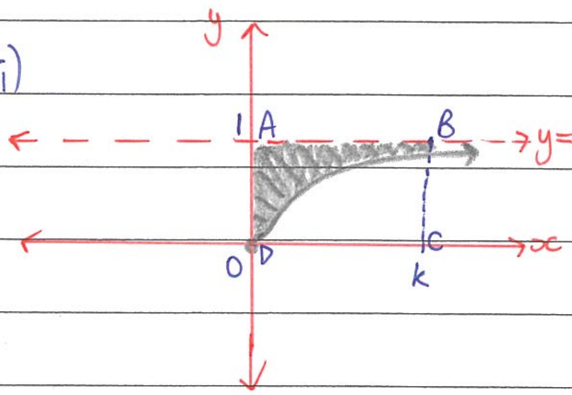
$\frac{dy}{dx} \neq 0$ as $e^x + e^{-x} \neq 0$ $\therefore$ No stationary points

Student Name: ____________________ Teacher Name: ____________________

14. c) (ii)



(iii)



Area of rectangle ABCD $= k \times 1$

$= k$

Area under curve $= \int_0^k \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

$= \left[\ln(e^x + e^{-x})\right]_0^k$

$= \ln(e^k + e^{-k}) - \ln(e^0 + e^0)$

$= \ln(e^k + e^{-k}) - \ln 2$

$\therefore$ Shaded area $= k - \left[\ln(e^k + e^{-k}) - \ln 2\right]$

$= k - \ln(e^k + e^{-k}) + \ln 2$

$= \ln e^k - \ln(e^k + e^{-k}) + \ln 2$

$= \ln\left(\frac{e^k}{e^k + e^{-k}}\right) + \ln 2$

$= \ln\left(\frac{2e^k}{e^k + e^{-k}}\right)$ as required.

Student Name: ______________________ Teacher Name: ______________________

14. c)(iv) Area $= \ln\left(\frac{e^k}{e^k+e^{-k}}\right) + \ln 2$

Now $\ln\left(\frac{e^k}{e^k+e^{-k}}\right) < 0$ as $\frac{e^k}{e^k+e^{-k}} < 1$

$\therefore$ Maximum value of area is always less than $\ln 2$ for all values of k.